EXERCISE FOR WEEK 7

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In the lecture on 19 October I introduced tensor products $V_1 \otimes \cdots \otimes V_k$ of more than one vector space, and defined the rank of tensors in such a space. I worked out the example of $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$, where almost all tensors have rank two, and I explained the connection with Cayley's hyper-determinant (see Wikipedia). Below I use the short-hand notation e_{ijk} for $e_i \otimes e_j \otimes e_k$.

Exercise 0.1. Let ω be the tensor

$$\omega := 3e_{111} + 4e_{121} + 2e_{211} + 3e_{221} + 2e_{112} + 3e_{122} + 1e_{212} + 2e_{222} \in \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2.$$

Write ω as a sum of as few pure tensors $u \otimes v \otimes w$ with $u, v, w \in \mathbb{C}^2$ as possible (and deduce the rank of ω).

Date: 20 October 2010.