EXTRA EXERCISE FOR WEEK 3

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Given a vector space V over $\mathbb{F}$, a linear map $\phi \in \mathcal{L}(V)$, and a field extension $\mathbb{K} \supseteq \mathbb{F}$, there is a unique $\mathbb{K}$-linear map $\phi_{\mathbb{K}} \in \mathcal{L}(V_{\mathbb{K}})$ such that $\phi_{\mathbb{K}} \circ \beta = \beta \circ \phi$, where $\beta : V \to V_{\mathbb{K}}$ is the natural $\mathbb{F}$-linear embedding from last week.

(1) Prove that the map $\mathcal{L}(V) \to \mathcal{L}(V_{\mathbb{K}})$, $\phi \mapsto \phi_{\mathbb{K}}$ is a homomorphism of $\mathbb{F}$-algebras.
(2) Prove that if $\phi_1, \ldots, \phi_k$ are $\mathbb{F}$-linearly independent elements of $\mathcal{L}(V)$, then $(\phi_1)_{\mathbb{K}}, \ldots, (\phi_k)_{\mathbb{K}}$ are $\mathbb{K}$-linearly independent elements of $\mathcal{L}(V_{\mathbb{K}})$.
(3) Prove that the minimal polynomial of $\phi_{\mathbb{K}}$ equals that of ϕ.

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