EXERCISES FOR WEEK 13

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In the lecture I discussed the Perron-Frobenius theorem.

Exercise 0.1. Another famous theorem is *Brouwer's fixed point theorem.* It says that, for every n, if f is a continuous map from the m-dimensional ball

$$B_m := \{x \in \mathbb{R}^m \mid ||x|| \leq 1\}$$

to itself, then there exists a point $x_0 \in B_m$ with $f(x_0) = x_0$.

(1) Prove the special case of Brouwer's theorem with $m = 1$.
(2) Deduce the following statement from Brouwer's fixed point theorem: if A is any non-negative matrix having no columns containing only zeroes, then there exists an eigenvector of A in $(\mathbb{R}_{\geq 0})^n$. (Hint: look at the case where $n = 2$ first; which value of m will you use?)

Exercise 0.2. Let G be a finite, undirected, connected, *bipartite graph*, that is, a graph in which the vertices can be coloured in two colours such that adjacent vertices have distinct colours. Let A be a the (symmetric) adjacency matrix of G, having a 1 at position (i, j) if i and j are connected by an edge, and zero otherwise. Let λ_0 be the Perron-Frobenius eigenvalue of A. Show that $-\lambda_0$ is also an eigenvalue of A.