EXERCISES FOR WEEK 11

JAN DRAISMA

In the lecture I discussed orthogonal, symplectic, and unitary groups; and exterior powers of vector spaces and of linear maps.

Exercise 0.1. Let f be the non-degenerate alternating form on $\mathbb{F}_3^4$ defined by

$$f(\mathbf{x}, \mathbf{y}) = x_1 y_2 - x_2 y_1 + x_3 y_4 - x_4 y_3;$$

here $\mathbb{F}_3$ is the field with three elements. Compute the number of elements of the symplectic group $\mathrm{Sp}(f)$. Hint: first recall how to count elements of GL_n over a finite field; then determine the conditions on the columns of a 4×4-matrix to lie in this symplectic group.

Exercise 0.2. Let V be a vector space and let $\phi : V \to V$ be a linear map. Let k be a positive integer less than or equal to $\dim V$.

(1) Prove that ϕ is invertible if and only if $\bigwedge^k \phi$ is invertible.
(2) Assume that ϕ is diagonalisable. Determine the eigenvalues of $\bigwedge^k \phi$ in terms of those of ϕ.