Invariant theory homework exercises
week 8

October 28, 2009

Exercise 1

Let G be the matrix group generated by $A, B \in \mathrm{GL}_2(\mathbb{C})$ given by

$$A := \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad B := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{1}$$

- Use Molien's theorem to prove that the Hilbert series of $\mathbb{C}[x,y]^G$ is given by $H(\mathbb{C}[x,y]^G, t) = \frac{1+t^6}{(1-t^4)^2}$.
- Find algebraically independent invariants f_1, f_2 of degree 4 and a third invariant f_3 of degree 6, such that $\mathbb{C}[x,y]^G = \mathbb{C}[f_1,f_2] \oplus \mathbb{C}[f_1,f_2]f_3$.

Exercise 2

There exists an even self-dual code $C \subseteq \mathbb{F}_2^{40}$, that contains no words of weight 4. How many words of weight 8 does C have?