Invariant theory homework exercises
week 5

October 8, 2009

1. Let $A \subset \mathbb{C}[x_1, \ldots, x_n]$ be a subalgebra. Let A_+ be the ideal in A consisting of all polynomials in A with zero constant coefficients. Show that if A_+ is a finitely generated ideal, then A is finitely generated as an algebra over $\mathbb{C}$.

2. Let G be a finite abelian group (written additively). Define the *Davenport constant* $\delta(G)$ to be the length m of the longest non-shortable expression $0 = g_1 + g_2 + \ldots + g_m, g_i \in G$. (*Non-shortable* means that no strict non-empty subset of the g_i's has sum zero.)

   Write $\beta(G)$ for the the minimal number m such that for every representation W of G the invariant ring $K[W]G$ is generated by the invariants of degree less or equal to m.

   Show that $\beta(G) = \delta(G)$ for any finite abelian group G.