Invariant theory homework exercises
week 4

September 29, 2009

1. Let V be a G-module and $\langle , \rangle$ a G-invariant inner product on V. Show that for any two non-isomorphic, irreducible submodules $V_1, V_2 \subset V$ we have $V_1 \perp V_2$, i.e. for all $v_1 \in V_1$, $v_2 \in V_2$ it holds that $\langle v_1, v_2 \rangle = 0$.

   Give an example where $V_1 \not\perp V_2$ when V_1 and V_2 are isomorphic as G-modules.

2. Let the symmetric group on 3 letters S_3 act on $\mathbb{C}[x_1, x_2, x_3]_2$ by permuting the variables. This action makes $\mathbb{C}[x_1, x_2, x_3]_2$ into a S_3-module. Give the decomposition of this module.

   Note that $\mathbb{C}[x_1, x_2, x_3]_2$ is the vector space of homogeneous polynomials of degree 2.