

Invariant theory with applications

assignment week 3

September 24, 2009

Exercise 1

Let $U \otimes V$ be the tensor product of the vector spaces U and V . Let u_1, \dots, u_s and u'_1, \dots, u'_t be two systems of linearly independent vectors in U and let v_1, \dots, v_s and v'_1, \dots, v'_t be two systems of linearly independent vectors in V . Suppose that

$$u_1 \otimes v_1 + \dots + u_s \otimes v_s = u'_1 \otimes v'_1 + \dots + u'_t \otimes v'_t. \quad (1)$$

Show that $s = t$.

Exercise 2

Let $T \in V_1 \otimes V_2 \otimes V_3$ be an element of the tensor product of V_1 , V_2 and V_3 . Suppose that there exist $v_1 \in V_1$, $v_3 \in V_3$, $T_{23} \in V_2 \otimes V_3$ and $T_{12} \in V_1 \otimes V_2$ such that

$$T = v_1 \otimes T_{23} = T_{12} \otimes v_3. \quad (2)$$

Show that there exist a $v_2 \in V_2$ such that $T = v_1 \otimes v_2 \otimes v_3$.