Invariant theory with applications
assignment week 1

September 10, 2009

Exercise 1

Let G be a finite group acting on $V = \mathbb{C}^n$, $n \geq 1$. Show that $\mathcal{O}(V)^G$ contains a nontrivial invariant. That is, $\mathcal{O}(V)^G \neq \mathbb{C}$. Give an example of an action of an infinite group G on V with the property that only the constant functions are invariant.

Exercise 2

Let $\rho : \mathbb{Z}/2\mathbb{Z} \to \mathrm{GL}_2(\mathbb{C})$ be the representation given by $\rho(1) := \left(\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$. Compute the invariant ring. That is, give a minimal set of generators for $\mathcal{O}(\mathbb{C}^2)^{\mathbb{Z}/2\mathbb{Z}}$.