HINTS FOR WEEK 5'S EXERCISES

(1) Please try to solve this exercise under the additional assmption that for every polynomial $f = f_0 + f_1 + \ldots + f_d \in A$, where f_i is a polynomial in which all terms have total degree i, the *homogeneous parts* f_i of f are all in A.

Bonus: prove or disprove the statement without this extra assumption.

(2) Here are some facts about finite Abelian groups G that you may want to use:
   (a) Every irreducible representation U of G is 1-dimensional. Hence for every $g \in G$ there is a unique number $\chi_U(g) \in \mathbb{C}^*$ such that $gu = \chi_U(g)u$ for all $u \in U$. We have $\chi_U(gh) = \chi_U(g)\chi_U(h)$, that is, χ is a homomorphism from G to $\mathbb{C}^*$. Conversely, every homomorphism $\chi : G \to \mathbb{C}^*$ gives rise to a one-dimensional representation of G.
   (b) The homomorphisms $\chi : G \to \mathbb{C}^*$ form an Abelian group $\check{G}$ with point-wise multiplication: $(\chi_1\chi_2)(g) := \chi_1(g)\chi_2(g)$.
   (c) The group $\check{G}$, called the *dual group* or *character group* of G, is isomorphic to G.