POSSIBLE SUBJECTS FOR CONCLUDING THE COURSE "DISCRETE STRUCTURES"

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Feit and Higman (1964): proved: If a finite, thick generalised n-gon exists, then $n \in \{2, 3, 4, 6, 8\}$. The proof uses eigenvalue techniques that nowadays permeate algebraic graph theory. Write an essay, in your own words, in which you prove this theorem.

Generalised Quadrangles: These are important rank-2 geometries, as we have seen. There are many results in the literature identifying GQs with certain properties as classical. Write an overview of such results, or an essay in which you prove some of them in detail.

Unitals: A *unital* is a $2-(n^3+1, n+1, 1)$-design. A classical construction for $n = q$ a prime power is as follows: take U equal to the set of points on the non-degenerate Hermitian quadric with equation $x^{q+1} + y^{q+1} + z^{q+1} = 0$ in $\mathrm{PG}(2, q^2)$ and as blocks all intersections of lines in $\mathrm{PG}(2, q^2)$ with U. There are many statements in the literature saying that under certain hypotheses unitals embedded into projective planes are classical. Search the literature for such results, and make an overview.

Polarities in (non-Desarguesian) planes: Recall that a polarity in a projective plane is a map σ interchanging points and lines preserving incidence, with the additional property that $\sigma^2 = \mathrm{Id}$. Write an essay in which you define some non-Desarguesian planes (e.g., mini-quaternion planes, see the bibliography on the web-site of the course) and explore their polarities.

Planes with a nice collineation group: Recall the definition of translation planes from the lecture notes: it is a projective plane having a line l_∞ such that for all $P \in l_\infty$ a central collineation with center P and axis l_∞ exists. Finite such planes are constructed from *spreads*. Consult the literature on spreads, translation planes, or other planes with nice collineation groups, and write an essay on one of these topics.

Matthieu/Steiner: Write an essay about the sequence of designs with parameters $2-(9, 3, 1)$, $3-(10, 4, 1)$, $4-(11, 5, 1)$, $5-(12, 6, 1)$ the beginning of which we have seen in the lecture, and their relation to the Matthieu group.

Small diagram geometries: Write an essay exploring geometries with diagrams consisting of 3 vertices and the various edge types occurring in the lecture notes.

Classical groups: During the course we studied various polar spaces, associated to sesquilinear forms. Write an essay on the associated groups leaving them invariant.