

APPLIED ALGEBRAIC GEOMETRY, SHEET 7

- (1) Let $A \in \mathbb{Z}_{\geq 0}^{m \times n}$ and let S be a Markov basis for A . Consider the ring homomorphism $\varphi : \mathbb{Z}[y_1, \dots, y_n] \rightarrow \mathbb{Z}[x_1, \dots, x_m]$ determined by $\varphi(y_j) = x^{Ae_j}$. Show that $\{y^{u_+} - y^{u_-} \mid u \in S\}$ generates $\ker \varphi$.
- (2) Check the last statement in the proof of Higman's lemma.

Handed out on November 5, to be handed in on November 11.