APPLIED ALGEBRAIC GEOMETRY, SHEET 6

(1) Find a Markov basis for the model of independence for d random variables taking values in $[r_1], \ldots, [r_d]$, that is, for the matrix $A \in \mathbb{Z}_{\geq 0}^{(r_1+\cdots+r_d)\times(r_1\cdots r_d)}$ that maps an $r_1 \times \cdots \times r_d$-table M of nonnegative integers to the tuple $(m_{+,\ldots,+,j,+,\ldots,+})_{i\in[d],j\in[r_i]}$ where the + indicates summing over all possible indices and the j is on the i-th position.

(2) The model of *no three way interaction* concerns three random variables X_1, X_2, X_3 that take values in $[r_1, r_2, r_3]$, respectively. It has design matrix A that maps an $r_1 \times r_2 \times r_3$-table M of nonnegative integers to the tuple $((m_{i,j,+})_{i,j}, (m_{i,+,k})_{i,k}, (m_{+,j,k})_{j,k})$. Play around with the program `4ti-2` to find Markov bases for specific instances. To do this, you proceed as follows:
   (a) install both `Mathematica` and `4ti2` on your computer;
   (b) run `Mathematica` on the file `no3way.m`, to produce a file `no3way.mat`;
   (c) run `4ti2-markov no3way`; this produces a file `no3way.mar`;
   (d) interpret the result.

   Can you guess Markov bases for $r_2 = r_3 = 2$ and r_1 running? What about $r_1 = r_2$ running and $r_3 = 2$?

Handed out on October 28, to be handed in on November 4.