APPLIED ALGEBRAIC GEOMETRY, SHEET 4

(1) Let V be a finite-dimensional vector space over $\mathbb{C}$ and let d be a nonnegative integer. Show that $\{v^d \mid v \in V\}$ linearly spans $S^d V$.

(2) Let V be the two-dimensional complex vector space with basis x, y. Determine the rank of the element

$$730x^6 - 1446x^5y + 1275x^4y^2 - 380x^3y^3 + 375x^2y^4 + 174xy^5 + 65y^6 \in S^6V$$

as well as a decomposition into that many terms.

(3) Let $V = \mathbb{C}^n$ with the standard symmetric bilinear form $(.|.)$, and let $d \in \mathbb{Z}_{\geq 2}$. Determine the dimension of the variety

$$\overline{\{v_1^d + \ldots + v_n^d \mid v_1, \ldots, v_d \in V \text{ and } (v_i|v_j) = \delta_{ij} \text{ for all } i, j\}} \subseteq S^d V.$$

These tensors are called *(complex-)orthogonally decomposable*.

Handed out on October 16, to be handed in on October 20.