A DIAMANT challenge

In $\mathbb{C}^n$ with standard Hermitian inner product

$$\langle \mathbf{u}, \mathbf{v} \rangle := \sum_{i=1}^{n} u_i \overline{v_i}$$

find as many vectors $\mathbf{u}_1, \ldots, \mathbf{u}_N$ as possible, subject to

- $\langle \mathbf{u}_i, \mathbf{u}_i \rangle = 1$ for all i, and
- $|\langle \mathbf{u}_i, \mathbf{u}_j \rangle| = \frac{1}{\sqrt{n+1}}$ for all $i \neq j$.

Reward: diamond-shaped paperweight!

Background

- name: SIC-POVM problem
- $N \leq n^2$
- up to dimension 45 and machine precision n^2 is attained
- even of special shape $\{A^i S^j u_0 \mid i, j = 0, \ldots, n-1\}$ for some $u_0 \in \mathbb{C}^n$

$$\zeta := e^{2\pi i/n} \quad A := \begin{bmatrix} 1 & & & \\ & \zeta & & \\ & & \ddots & \\ & & & \zeta^{n-1} \end{bmatrix} \text{ and } S := \begin{bmatrix} & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \\ 1 & & & & \end{bmatrix}$$