

## A DIAMANT challenge

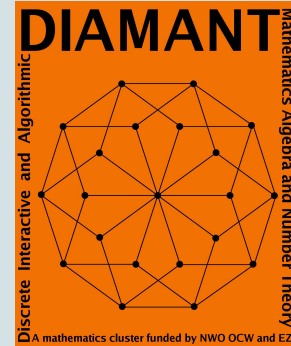
In  $\mathbb{C}^n$  with standard Hermitian inner product

$$\langle \mathbf{u}, \mathbf{v} \rangle := \sum_{i=1}^n u_i \overline{v_i}$$

find as many vectors  $\mathbf{u}_1, \dots, \mathbf{u}_N$  as possible,  
subject to

- $\langle \mathbf{u}_i, \mathbf{u}_i \rangle = 1$  for all  $i$ , and
- $|\langle \mathbf{u}_i, \mathbf{u}_j \rangle| = \frac{1}{\sqrt{n+1}}$  for all  $i \neq j$ .

**Reward:** diamond-shaped paperweight!



## Background

- name: SIC-POVM problem
- $N \leq n^2$
- up to dimension 45 and machine precision  $n^2$  is attained
- even of special shape  $\{A^i S^j u_0 \mid i, j = 0, \dots, n-1\}$  for some  $u_0 \in \mathbb{C}^n$

$$\zeta := e^{2\pi i/n} \quad A := \begin{bmatrix} 1 & & & \\ & \zeta & & \\ & & \ddots & \\ & & & \zeta^{n-1} \end{bmatrix} \quad \text{and} \quad S := \begin{bmatrix} & 1 & & \\ & & 1 & \\ & & & \ddots \\ 1 & & & & 1 \end{bmatrix}$$